

Rapidity Dependence of Elliptic Flow from Hydrodynamics

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Outline

- Introduction
- Results so far
- T^{th} dependence of $v_2(\eta)$
- Initial condition from the CGC and its consequence
- Summary & discussion

Introduction

$v_2(y)$ is supposed to reflect global dynamics...

→ How obtain by hydrodynamics?

Non-central coll. → No cylindrical sym.

Non-Bjorken behavior → No scaling ansatz

High energy collisions (@RHIC) → No Cartesian coordinate?



Need full 3D hydro
simulations in τ - η coordinate

T.H., PRC65,011901(2002).

From Experimental Point of View

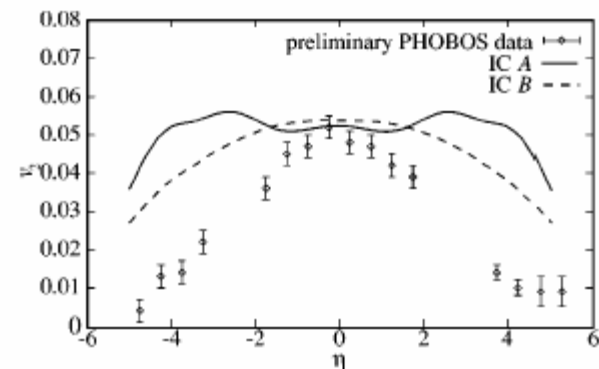
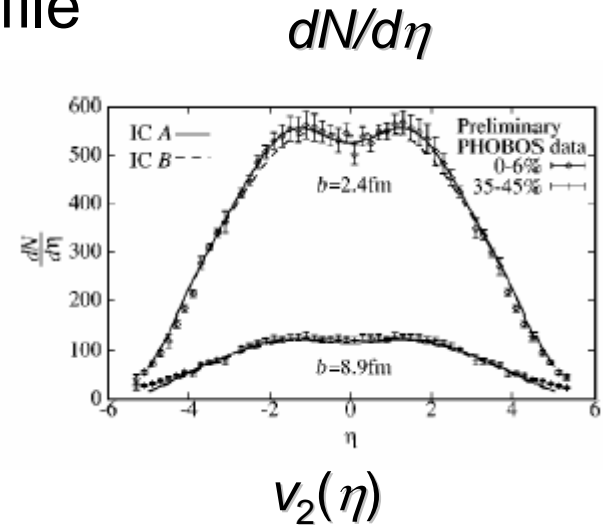
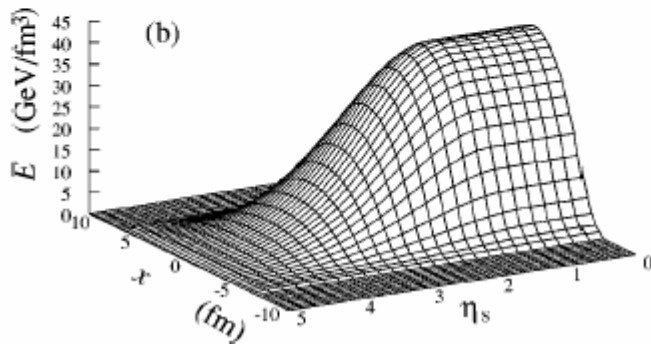
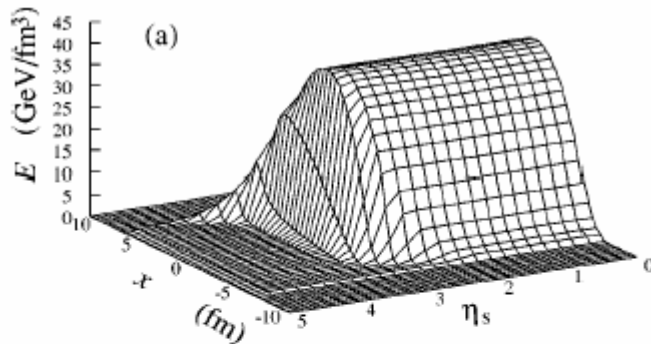
Need broad acceptance in longitudinal direction

See S.Manly's talk

Results so far (1)

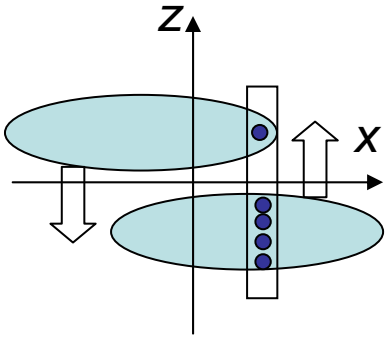
T.H., PRC65,011901(2002).

- The shape of $v_2(\eta)$ depends largely on initial longitudinal profile

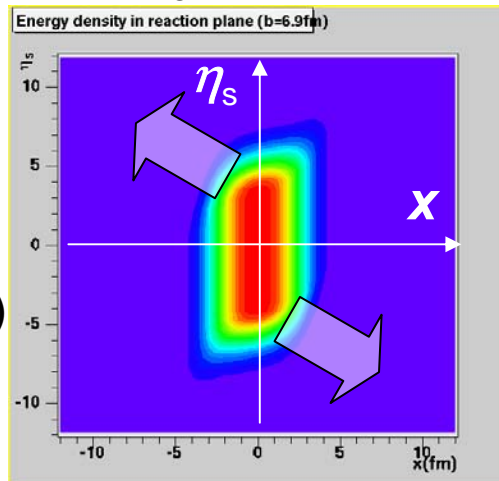


Local Rapidity Shift

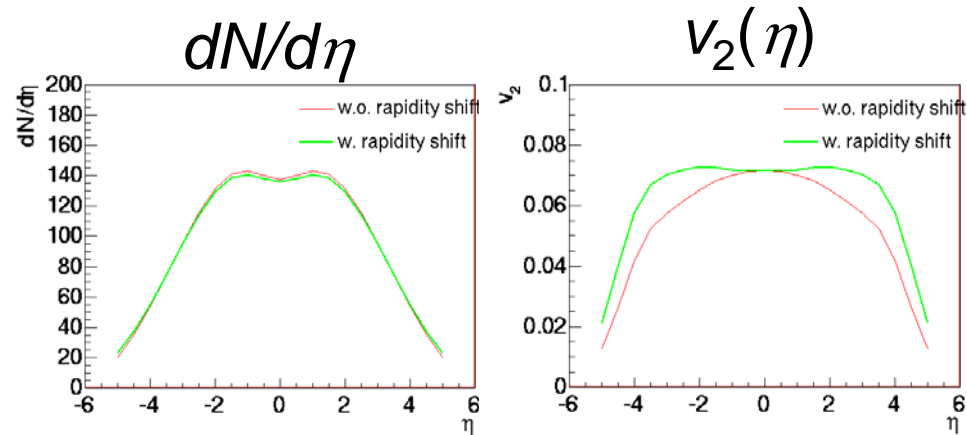
- Local rapidity shift
(J.Sollfrank *et al.*, Eur.Phys.J.C6,525(1999))



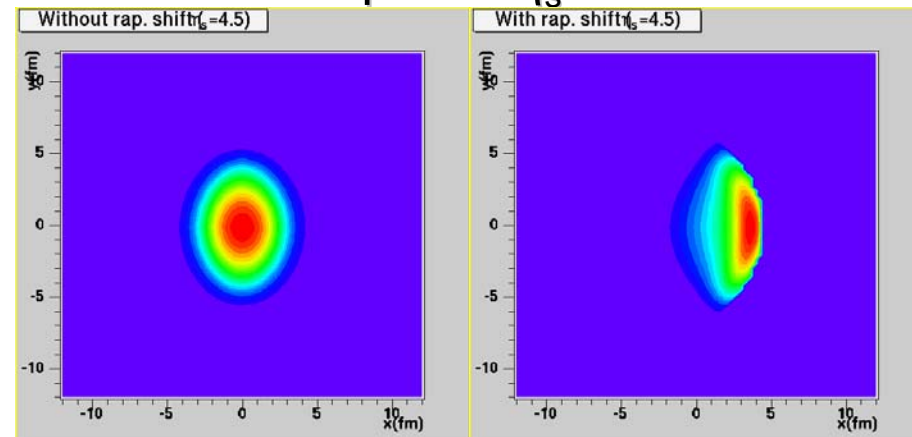
$E(x, \eta_s)$ at $b=6.9\text{fm}$



Direct charged, $T^{\text{th}}=140\text{MeV}$, $b=6.9\text{fm}$



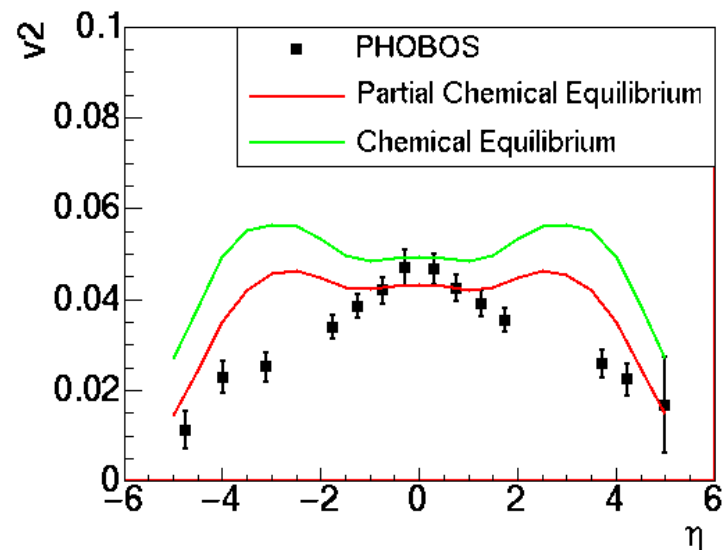
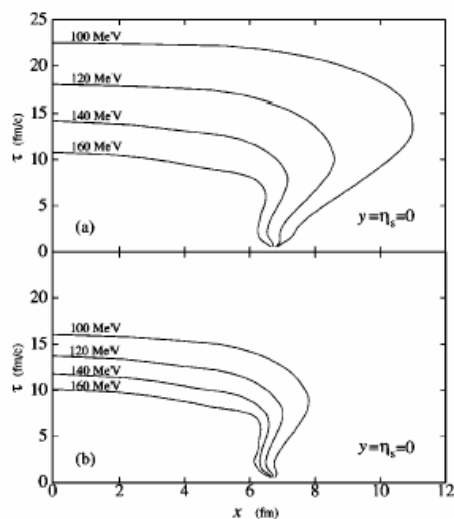
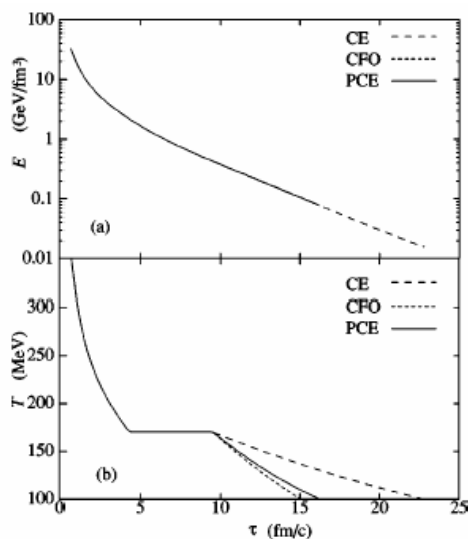
“Shape” at $\eta_s=4.5$



Third flow?
(L.P.Csernai and
D.Rohrich, PL
B458, 454(1999))

Results so far (2) T.H. and K.Tsuda, PRC66,054905(2002).

- v_2 is reduced by chemical non-equilibrium property



- Space-time evolution is completely different from conventional (chemically equilibrated) EOS.

$v_2(\eta)$ and $v_2(y)$

P. Kolb, Heavy Ion Phys. **15**, 279(2002).

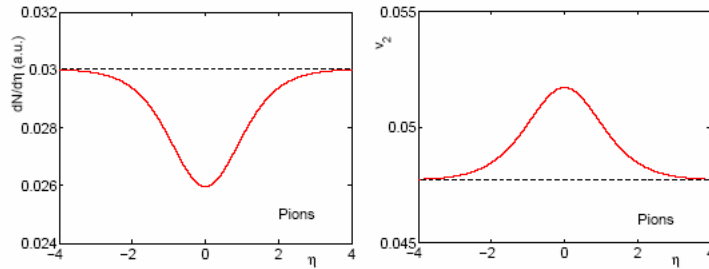
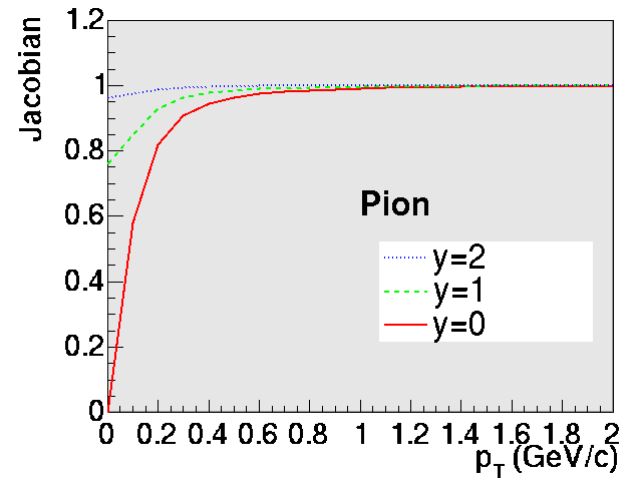


Fig. 6. Transformation of boost-invariant quantities (independent of rapidity y) to pseudo-rapidity. The left plot shows the effect of the Jacobian of the transformation on the particle spectra, the right plot the influence on the elliptic flow coefficient v_2 .

Jacobian as an weight fn.

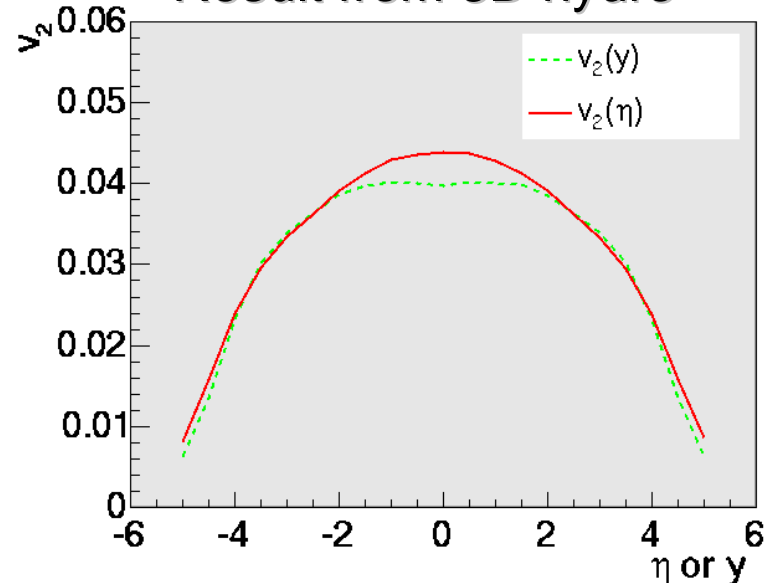


Jacobian between y and η

$$J = p/E = \sqrt{1 - \frac{m^2}{(p_T^2 + m^2) \cosh^2 y}}$$

$$\begin{aligned} \langle \cos(2\phi) \rangle &= \frac{\int d^2 p_T \cos(2\phi) (dN/d^2 p_T d\eta)}{\int d^2 p_T (dN/d^2 p_T d\eta)} \\ &= \frac{\int d^2 p_T \cos(2\phi) (J \times dN/d^2 p_T dy)}{\int d^2 p_T (J \times dN/d^2 p_T dy)} \end{aligned}$$

Result from 3D hydro

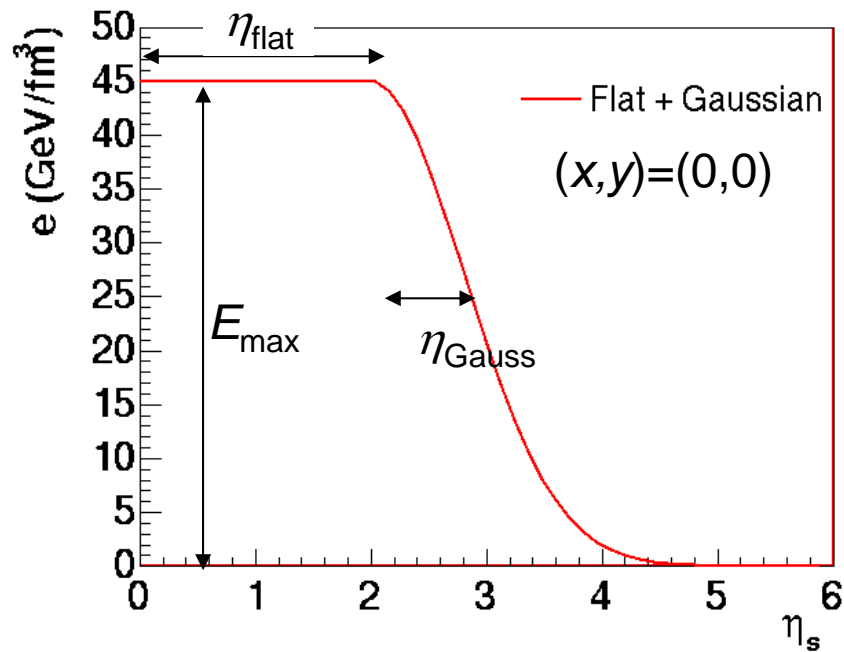


Summary (part 1)

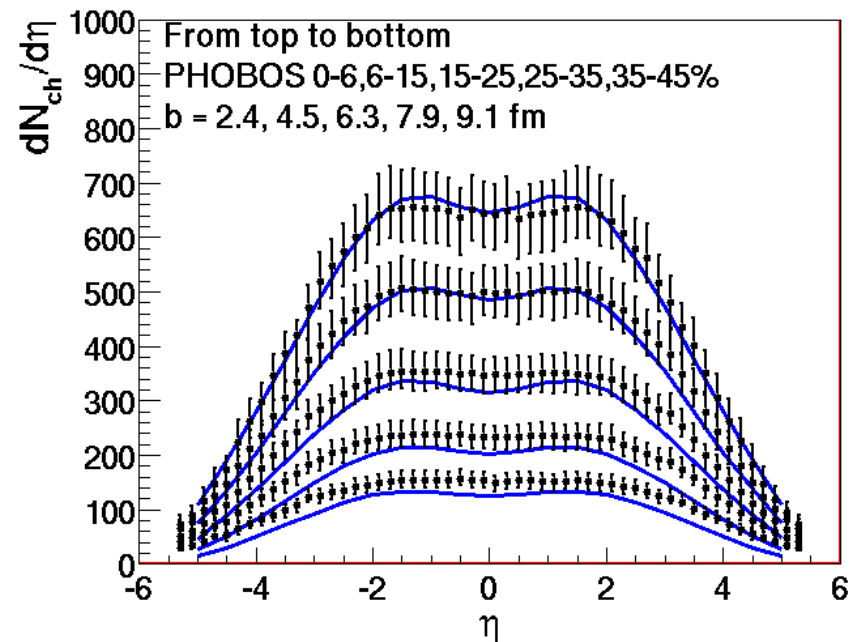
- Ambiguity of init. condition.
 - Smaller flat region would be good.
 - No local rapidity shift
- Realistic EOS (chemically non-eq.)

Return game in 200 GeV collisions

Initial Condition in Long. Direction

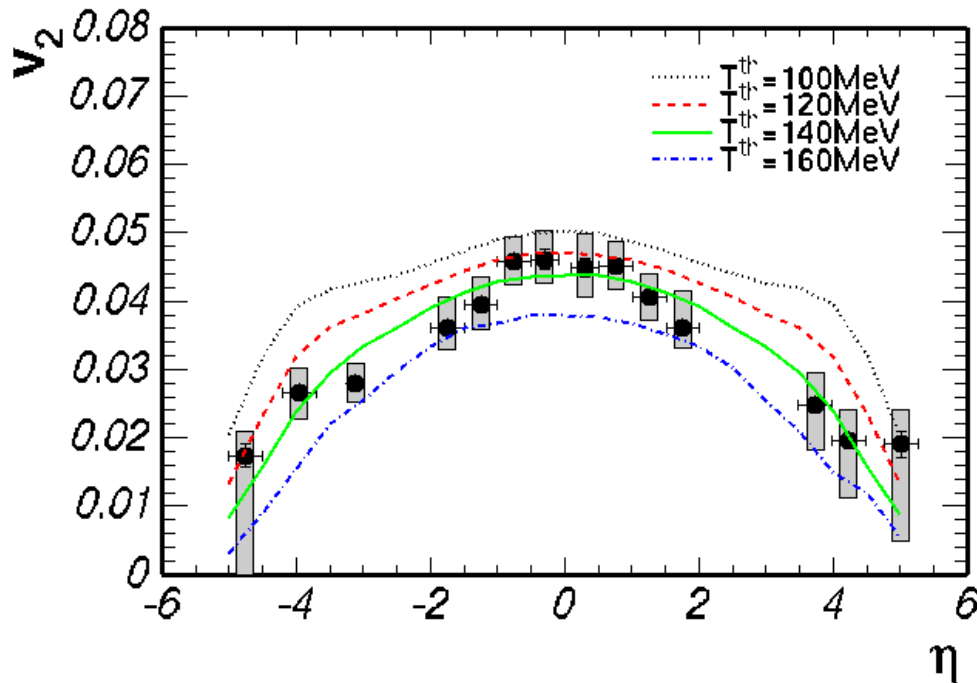


- $E_{\max} = 45$ GeV/fm³
- $\eta_{\text{flat}} = 2.0$
- $\eta_{\text{Gauss}} = 0.8$
- ★ No local rapidity shift



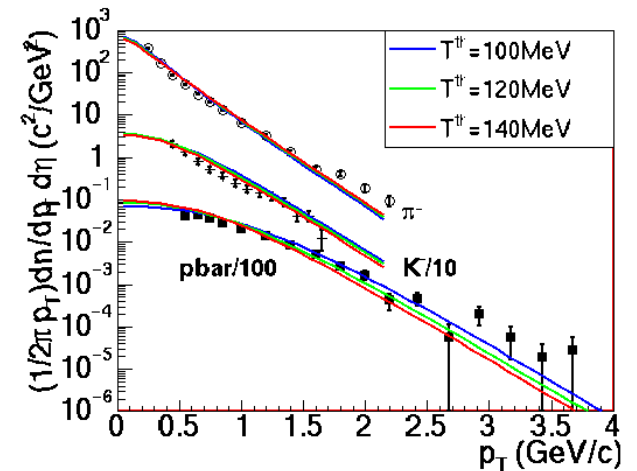
of binary coll. scaling for finite b.
Works well as centrality dep. for
initial condition.
(P.Kolb *et al.*, Nucl.Phys.A696,197,(2001))

T^{th} Dependence of v_2



- v_2 grows also in hadron phase.
- Eventually, $v_2(y)$ becomes flat as T^{th} decreases

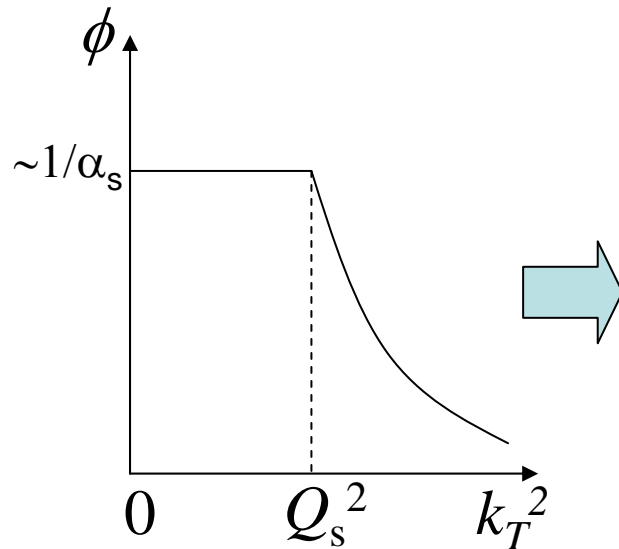
As far as pions, T^{th} is not determined by p_{T} slope within chemically non-equilibrium EOS model.



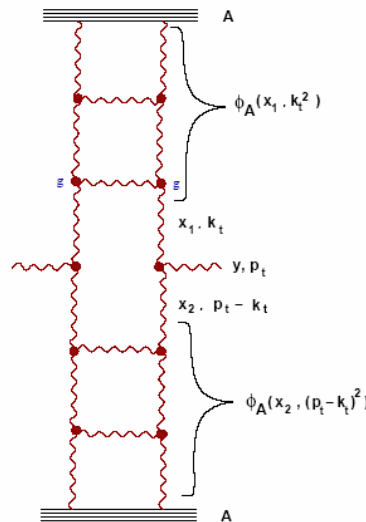
$dN/d\eta$ from a Saturation Model

D. Kharzeev and E. Levin, Phys.Lett.B523,79(2001)

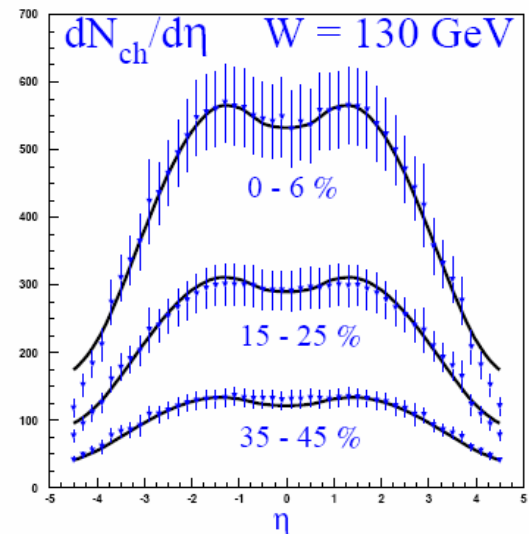
$$xG_A(x, Q^2) = \int^{Q^2} dk_T^2 \varphi_A(x, k_T^2)$$



$gg \rightarrow g$



Parton-hadron duality



$$E \frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2 - 1} \frac{1}{p_T^2} \int dk_T^2 \alpha_s \varphi_A(x_1, k_T^2) \varphi_A(x_2, (p - k)_T^2)$$

Initial Condition from CGC (hydrodynamic afterburner for CGC)

Saturation scale at a transverse position:

$$Q_s^2(x_\perp) = \frac{4\pi^2 N_c}{N_c^2 - 1} \alpha_s(Q_s^2) x G(x, Q_s^2) \frac{\rho_{\text{part}}(x_\perp)}{2}$$

where

$$x G(x, Q^2) = K \ln \left(\frac{Q^2 + \Lambda^2}{\Lambda_{\text{QCD}}^2} \right) x^{-\lambda} (1-x)^n$$

Unintegrated gluon distribution can be written

$$\phi(x, k_T^2) = \begin{cases} \frac{\kappa}{\alpha_s(Q_s^2)} \frac{Q_s^2}{Q_s^2 + \Lambda^2}, & k_T \leq Q_s(x), \\ \frac{\kappa}{\alpha_s(Q_s^2)} \frac{Q_s^2}{k_T^2 + \Lambda^2}, & k_T > Q_s(x). \end{cases}$$

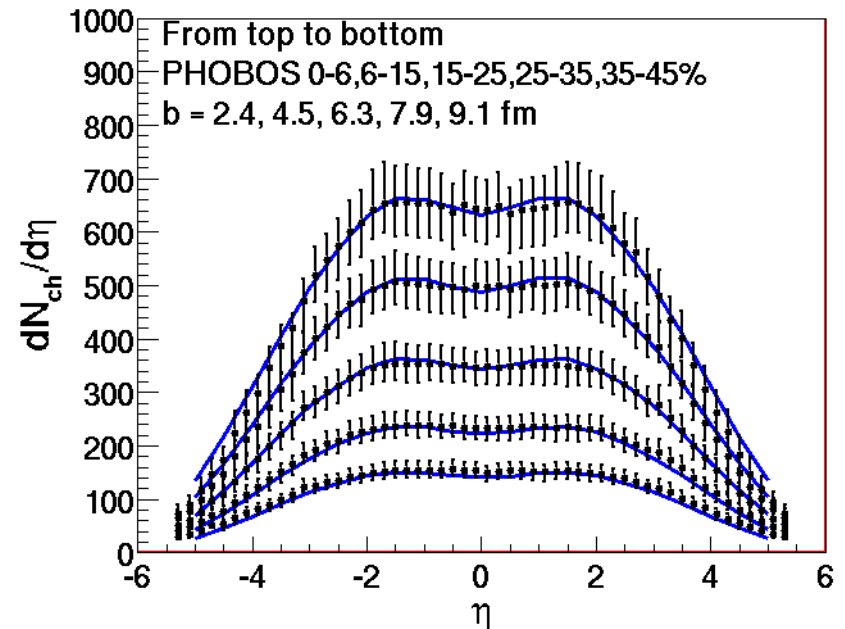
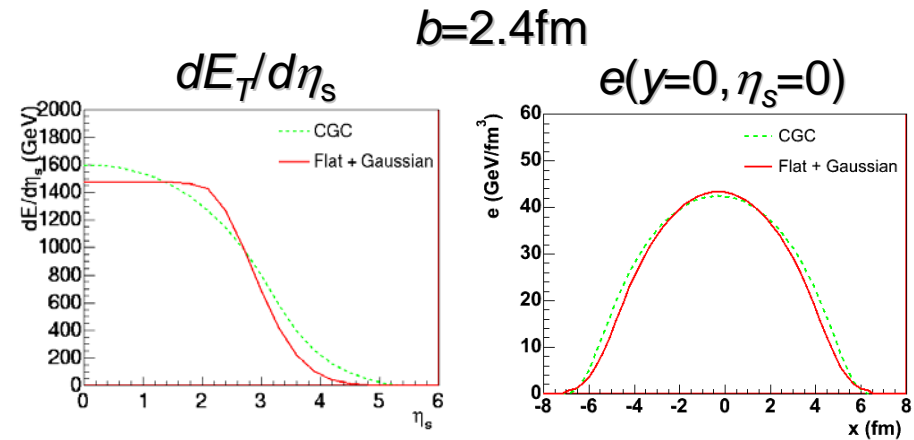
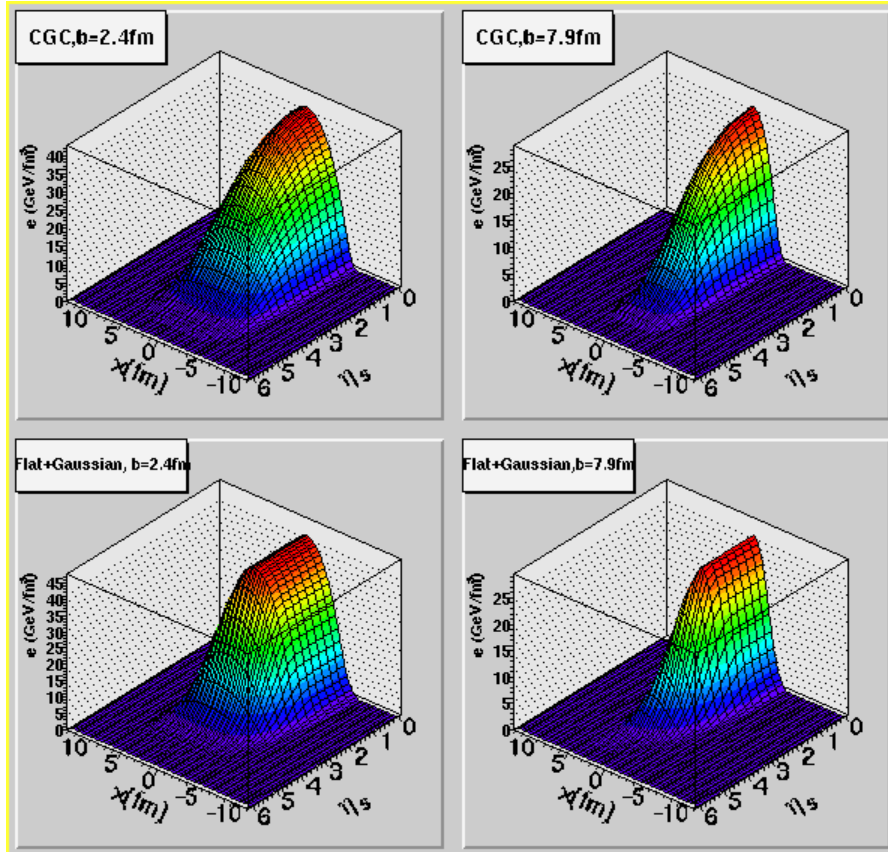
$$\begin{aligned} K &= 0.7 \\ \kappa^2 &= 0.94 \quad (T^{\text{th}} = 100 \text{ MeV}) \\ \lambda &= 0.2 \end{aligned}$$

$$\frac{dE_g}{d^2x_\perp dy} = \frac{4N_c}{N_c^2 - 1} \int p_T \frac{d^2 p_T}{p_T^2} \int d^2 k_T \alpha_s \varphi_A(x_1, k_T^2; x_\perp) \varphi_A(x_2, (p - k)_T^2; x_\perp)$$

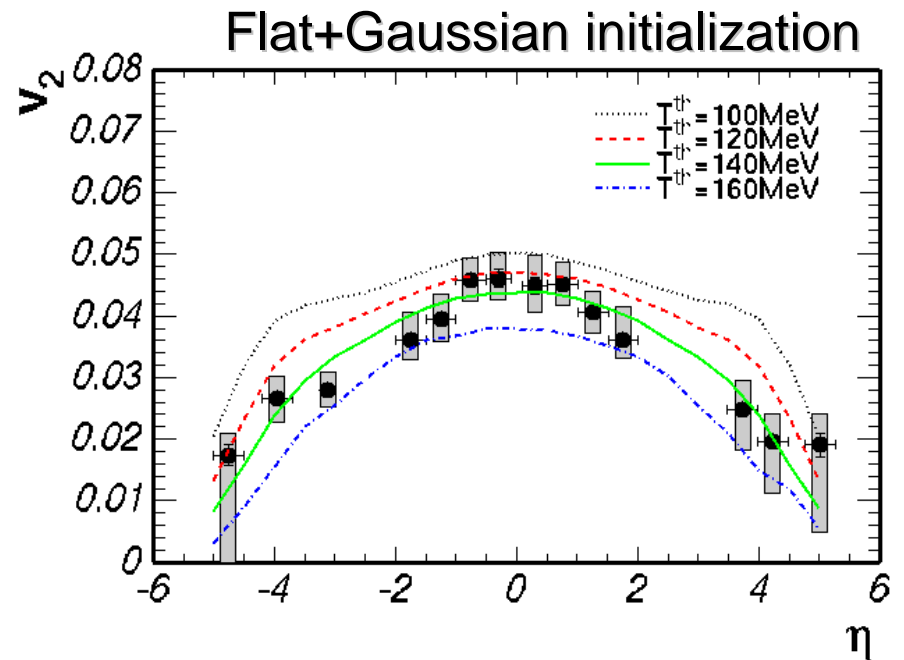
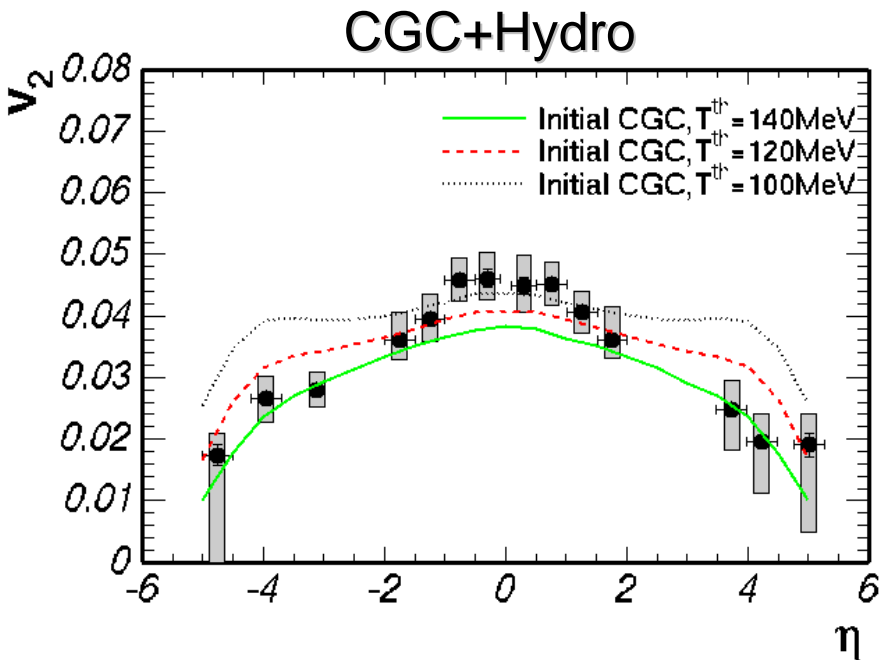
Momentum rapidity $y \rightarrow$ space time rapidity η_s

$$e(\tau_0, \vec{x}) = \frac{dE_g}{\tau_0 d\eta_s dx_\perp^2} \quad \longrightarrow \quad \begin{array}{l} \text{Input for hydrodynamics} \\ \text{"CGC + Hydro (+ Jet) model"} \end{array}$$

Initial Energy Density Distribution and $dN_{ch}/d\eta$



$v_2(\eta)$ from CGC+Hydro

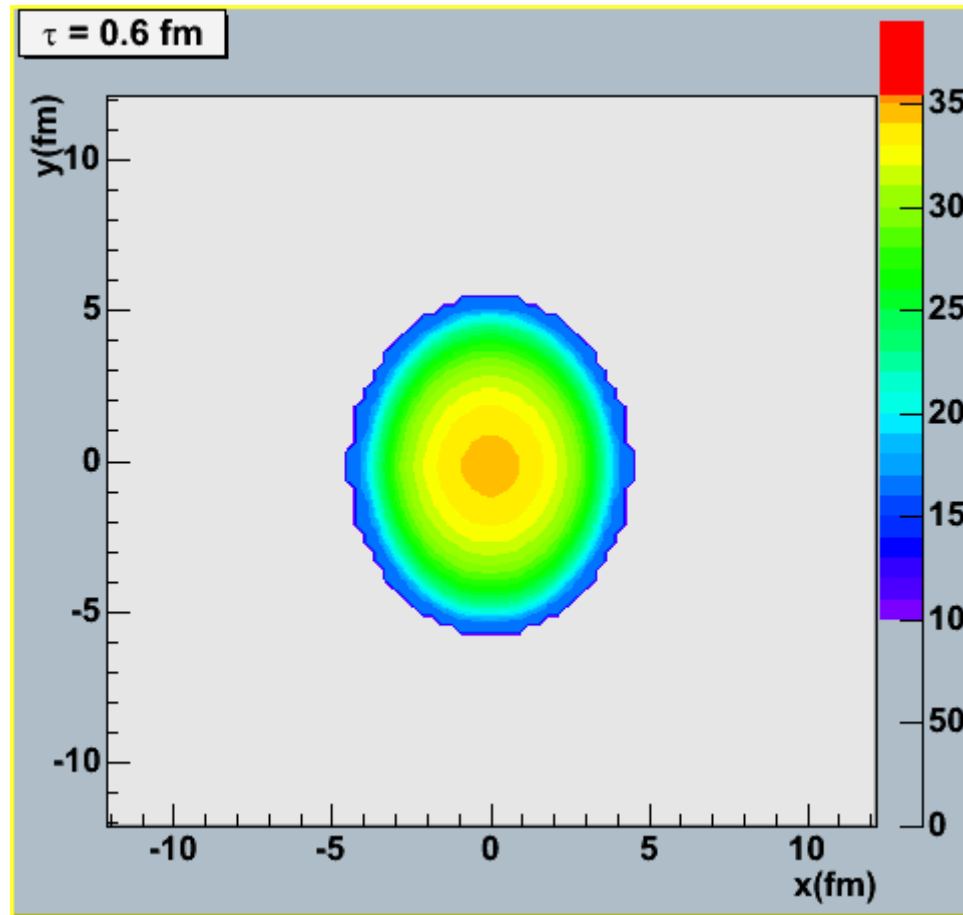


- Slightly suppressed near midrapidity region
 - Pressure gradient in longitudinal direction
 - Pressure gradient in transverse direction
 - Stronger longitudinal flow and weaker transverse flow in CGC+hydro case than in flat+Gaussian case.

Summary & Discussion

- There IS a solution for $v_2(\eta)$ from hydrodynamics.
- Further systematic studies are needed.
 - Initial condition? T^{th} dependence ? EOS ?
- CGC+hydro(+jet) model (“Improvement” of I.C.)
- Consistency check
 - $\langle p_T \rangle(y)$ & p_T spectra in forward rapidity (BRAHMS data)
 - $v_2(p_T)$ @ forward rapidity region ?
 - $v_1(y)$??? ← third flow components ?

Thank you!

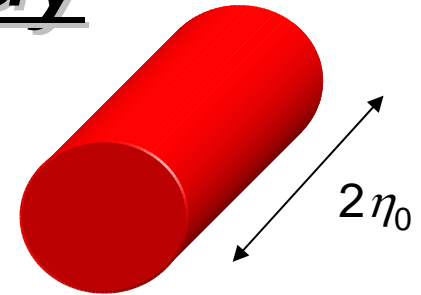


CGC+hydro at $b=6.9\text{fm}$ in transverse plane

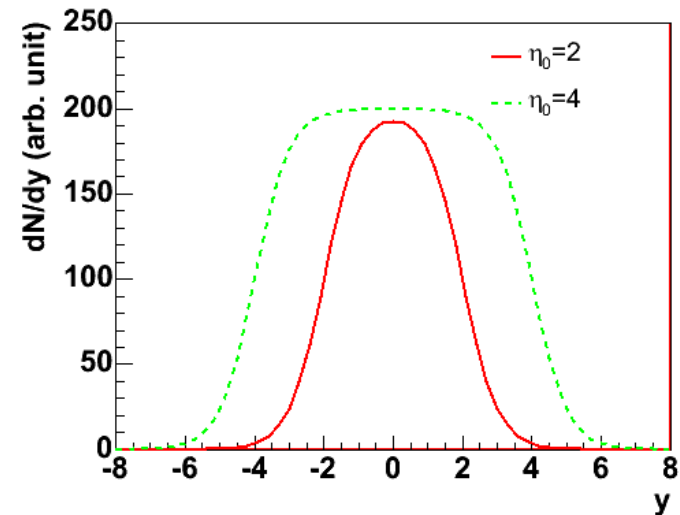
Flat Energy \neq Flat dN/dy

Basic assumption

1. Finite Bjorken rod ($-\eta_0 < \eta_s < \eta_0$)
2. Massless pions
3. Constant T and Boltzmann approximation



$$\begin{aligned}
 \frac{dN}{dY} &\propto \pi R^2 \int_{-\eta_0}^{\eta_0} d\eta_s \int_0^\infty p_T dp_T \exp(-p_T \cosh(\eta_s - Y)/T) \\
 &= \pi R^2 \int_{-\eta_0}^{\eta_0} d\eta_s \frac{T^2}{\cosh^2(\eta_s - Y)} \\
 &= \pi R^2 T^2 \int_{\tanh(-\eta_0 - Y)}^{\tanh(\eta_0 - Y)} dx \\
 &= \pi R^2 T^2 (\tanh(Y + \eta_0) - \tanh(Y - \eta_0))
 \end{aligned}$$



Accepted Events by PHOBOS

Not minimus bias!

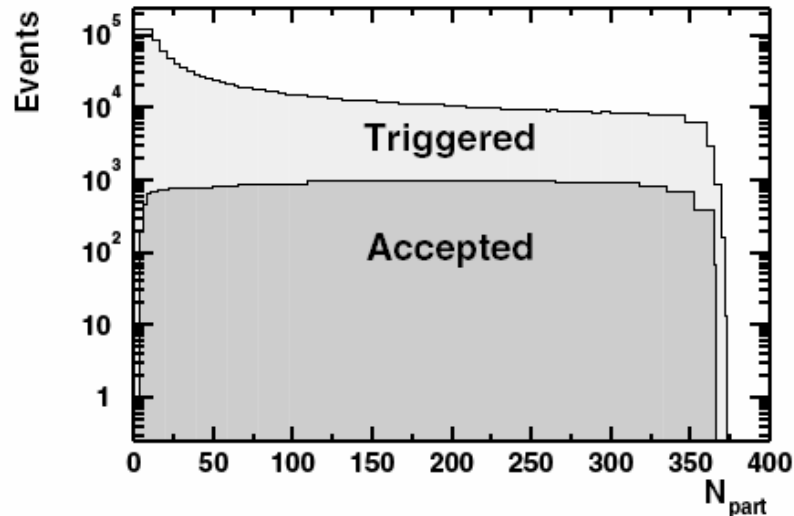
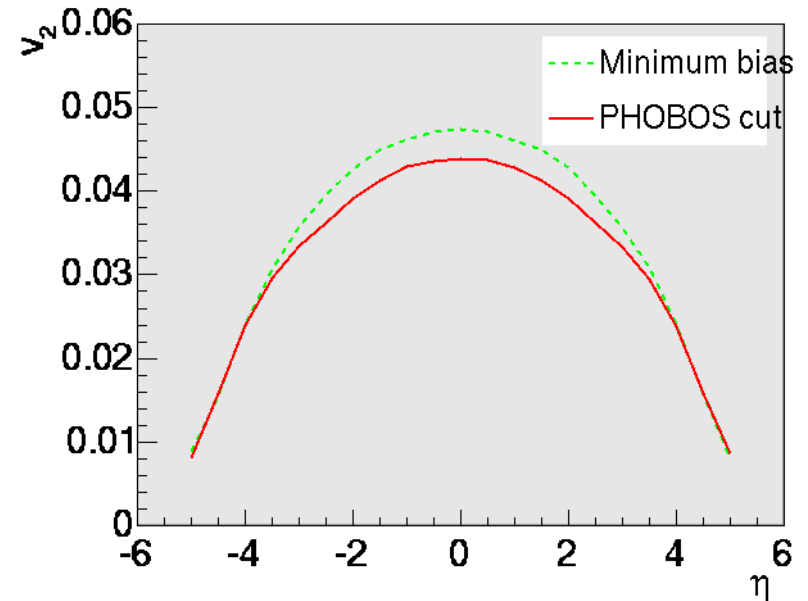


FIG. 1. The event distribution as a function of N_{part} for triggered events (upper curve) and for data accepted for use in the final analysis (lower curve).

B.B.Back et al.(PHOBOS), PRL89,222301(2002).

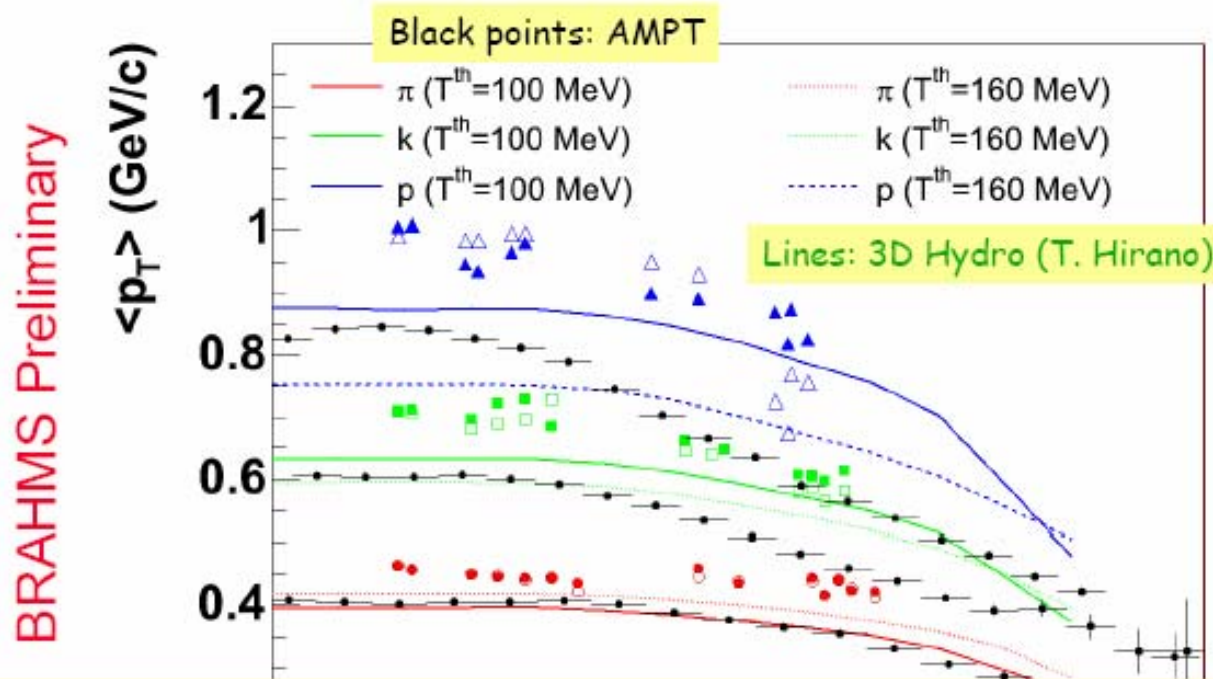


In hydro, average over N_{part}
in multiples of 25 up to 325.

Mean p_T as a Function of y



$\langle p_T \rangle$ vs rapidity



- AMPT (HIJING + Re-scattering) shows stronger y dependence than data
- 3D-Hydro describe y -dependence of data with a single T_{th} value at ~ 100 MeV (initial condition is tuned for $dN/d\eta$ and $T_{ch} = 170$ MeV)
- Strong Radial Flow in $0 < y < 3$ (Blast-Wave Fit gives $T \sim 120$ MeV $\beta \sim 0.6$ at $y=0$)

Blast Wave Fit in Forward Rapidity Region by BRAHMS

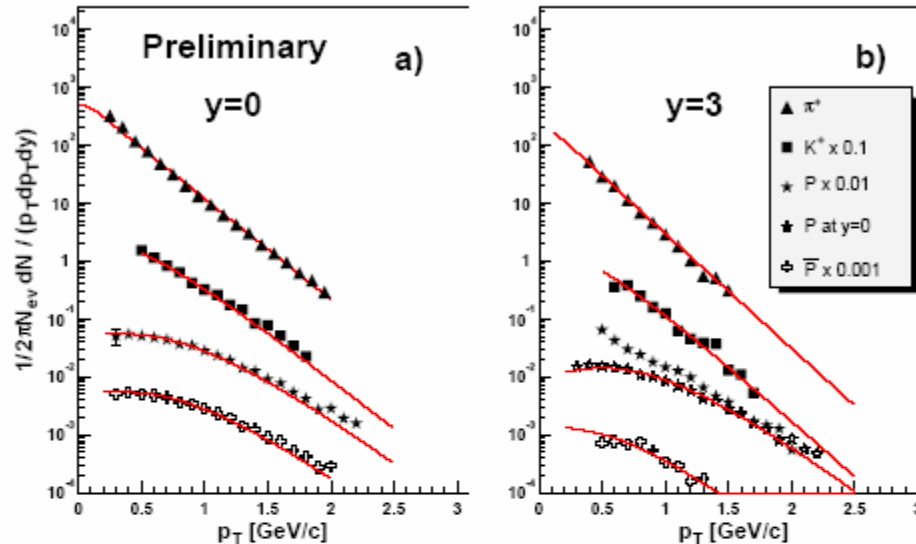


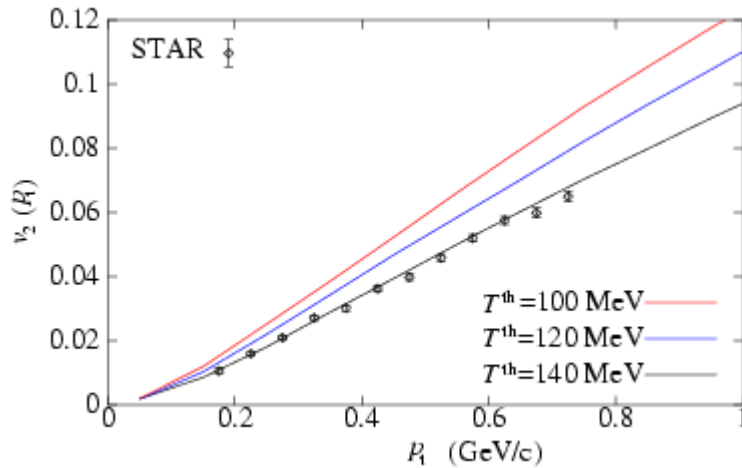
FIGURE 2. “Blast wave” fits to pions, kaons and protons at rapidity $y=0$ (panel a) and $y=3$ (panel b)

TABLE 1. Results of blast wave fits

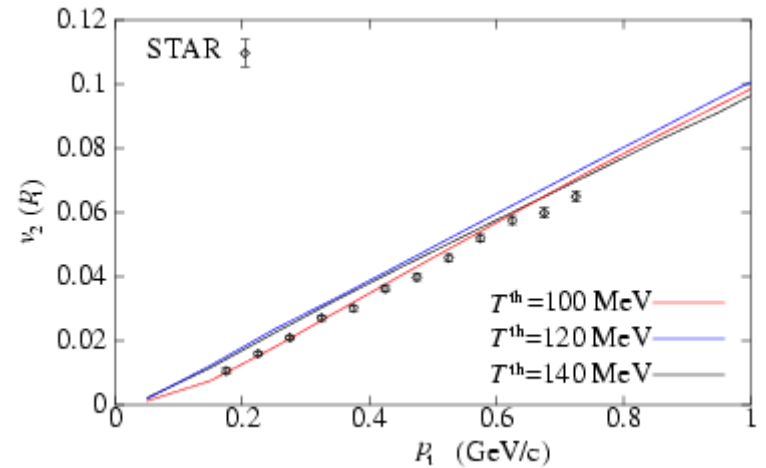
Rapidity	Temperature [MeV]	Velocity
0	127 ± 2	0.57 ± 0.01
0.7	112 ± 1	0.60 ± 0.01
2.2	128 ± 3	0.50 ± 0.01
3	136 ± 4	0.44 ± 0.02

R.Debbie (BRAHMS), proceeding for The 8th Conference on Intersections of Particle And Nuclear Physics (CIPANP2003), New York City, New York (May 19-24, 2003).

$v_2(p_T)$ @ 130A GeV



Chemical non-eq. model



Chemical eq. model

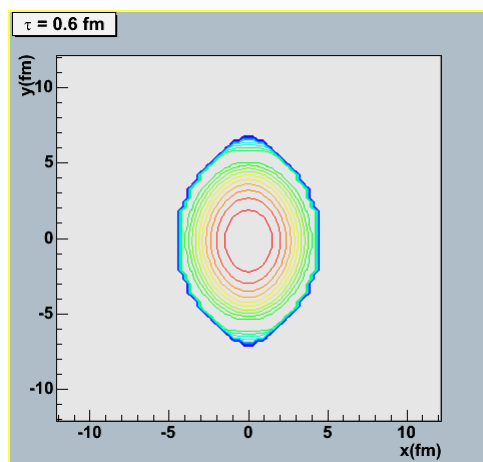
Table. T^{th} dependence for pions

	p_T slope	$v_2(p_T)$
Chem. eq.	yes	no
Chem. non-eq.	no	yes

Freezeout Hypersurface



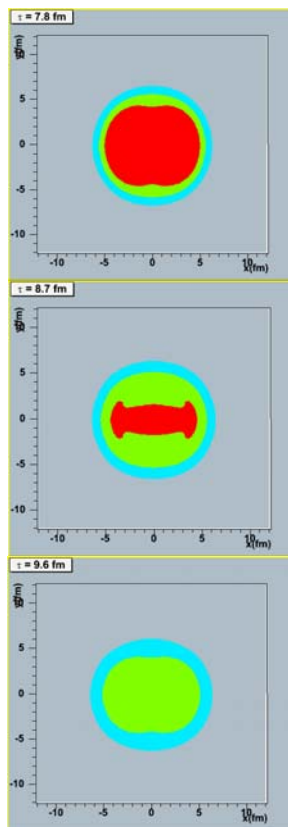
Initial temperature ($b=6.9\text{fm}$)



PCE

τ (fm)

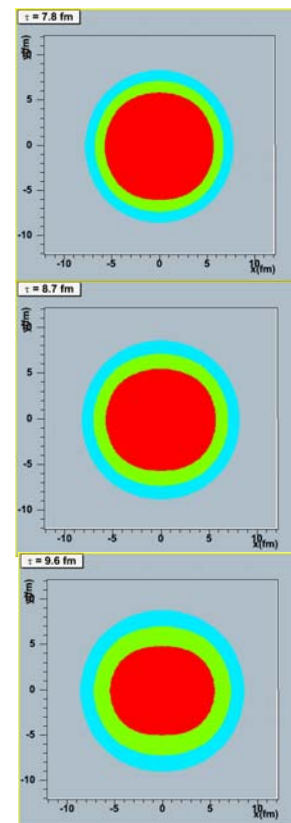
CE



7.8

8.7

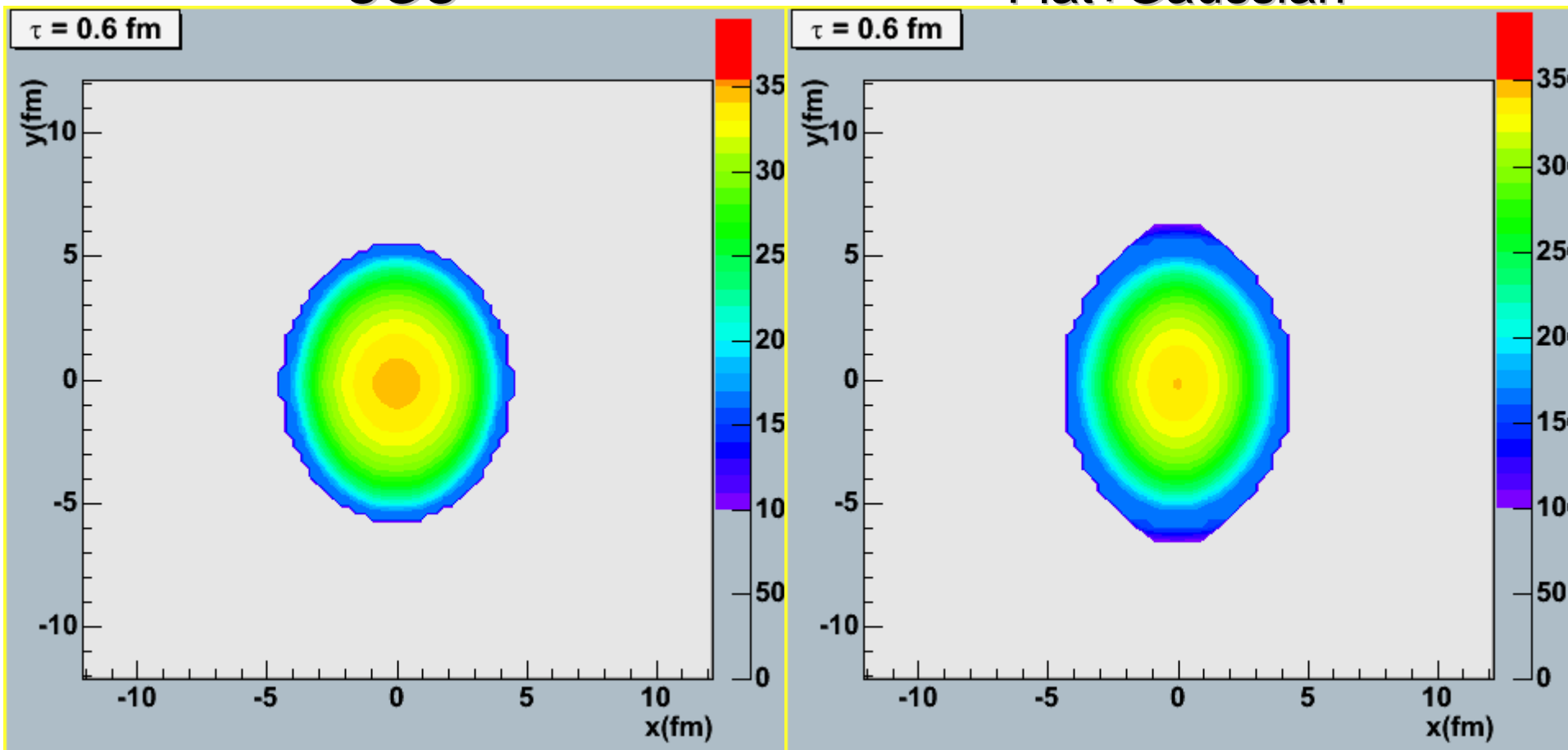
9.6



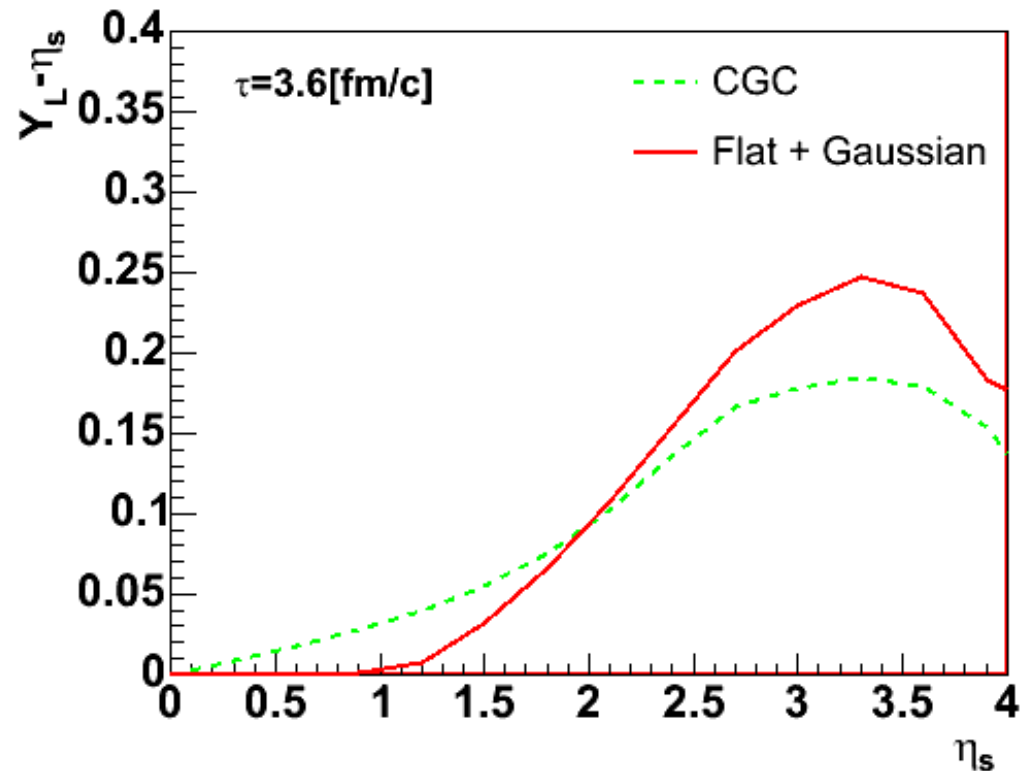
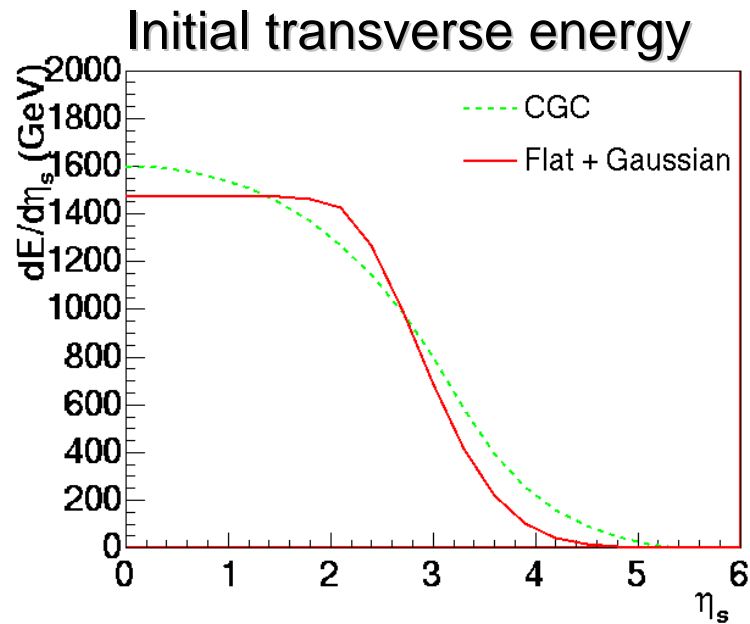
Movie

CGC

Flat+Gaussian



Longitudinal Acceleration



$Y_L - \eta_s$: Deviation from scaling flow